Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 2, pp. 256–263, 2002

WAVE-FIELD STRUCTURE IN ACTIVE BUBBLE SYSTEMS IN SHOCK TUBES WITH "DISCONTINUITIES" IN CROSS SECTION

V. K. Kedrinskii, I. V. Maslov, and S. P. Taratuta

UDC 532.529+532.593

The wave structure in active bubble media in shock tubes with sudden changes of profiles in the form of "discontinuities" in cross section and a one-phase liquid waveguide is analyzed numerically. In axisymmetric formulation, the paper studies wave amplification due to reflection from a wall and focusing at the butt-end of a rigid rod aligned coaxially with the channel. In this configuration, the amplification effect results from two-dimensional cumulation of the shock wave after it leaves the annular channel and reaches the butt-end of the rod. A Mach configuration forms in the focus spot. The geometrical characteristics of the shock tube allow one to control (to some extent) the amplification coefficient and the coordinates of the focus spot. In particular, it is shown that the wave can be focused near the second discontinuity of cross section — a rigid wall (in the region of passage through the interface to the waveguide) — and intensified upon reflection. If the waveguide radius is equal to the height of the Mach stem, the reflected wave has a maximum amplitude.

Introduction. As is known, in bubble media, shock waves interacting with these media can be amplified during propagation [1–3], collision with a rigid wall, or focussing [4]. In particular, Kedrinskii et al. [4] showed that in these processes, wave amplification depends strongly on the gas-phase volume fraction k_0. For example, for $k_0 = 0.01$, due to collision of identical shock waves, their amplitude increases by an order of magnitude. It has been shown [4, 5] that both cavitating and bubble systems with a passive gas phase and explosive gas mixtures can be regarded as active media able to absorb external disturbance energy, amplify this disturbance, and then reradiate it as an acoustic pulse. Studies of these systems in designing hydroacoustic analogs of laser systems ("hydroacoustic lasers") involves the problem of transmission of the acoustic pulse generated by the system into a liquid with the least losses. A solution of this problem was proposed in [5], where the interaction of a plane shock wave with a spherical passive bubble cluster was studied and it was shown that the wave refracted into the cluster can be focused in the vicinity of the cluster–fluid interface.

The present paper analyzes special features of the wave structure, mainly, in active bubble systems in shock tubes with sudden changes in cross section and with a one-phase liquid waveguide.

Formulation of the Problem. Sudden changes in the cross section of a shock tube of radius $R_{\rm ST}$ (Fig. 1) are produced by changes in shock-tube profile (transition from region 3 to region 5) or/and by an inside coaxial rigid cylinder 2 (radius $r_{\rm cyl}$ and length $L_{\rm cyl}$), which forms an annular channel 1 filled with a two-phase mixture similarly to region (3). The condition $L_{\rm SW} \leqslant L_{\rm cyl}$ is satisfied ($L_{\rm SW}$ is the characteristic distance at which the steady-wave regime initiated at the left butt-end of the shock tube is established). This inner geometry of the channel stimulates the effects of wave collision and focusing (the pressure in region 3 is changed by varying the distance L between the butt-end and the wall) in the bubble medium on intensification of the acoustic pulse generated by waveguide 5 (radius $r_{\rm out}$). The characteristic profile of the wave generated by the bubble medium into the waveguide is calculated at cross section 6 at a distance $L_{\rm WG}$ from the entrance (the interface between regions 3 and 5).

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 2, pp. 101–109, March–April, 2002. Original article submitted November 26, 2001.

0021-8944/02/4302-0256 $27.00

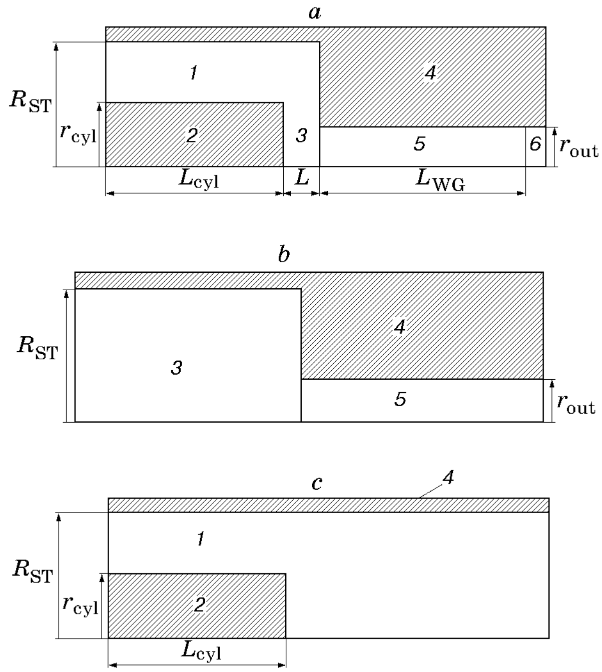

Fig. 1. Geometry of the shock-tube channel: combination of sudden enlargement and sudden contraction of the channel (a), sudden contraction of the channel (b), and sudden enlargement of the channel (c); 1) annular channel; 2) rigid rod; 3) working section; 4) shock-tube wall; 5) waveguide; 6) control waveguide cross section.

Two-Phase Model. To describe wave processes in bubble hydrodynamic shock tubes, we use a model of mathematical physics based on the modified Iordansky–Kogarko–van Wijngaarden model [6]. For the reactive gas phase, this model is supplemented with a chemical-reaction kinetics equation of the type of the Todes equation [6], a more general kinetic equation [7], or the simple condition of an instantaneous adiabatic explosion in a constant volume. An analysis of various approaches to describing wave processes in active media with bubbles filled with an explosive gas mixture showed that a simplified formulation can be used, ignoring the reaction kinetics in bubbles during their compression by shock waves [8]. This formulation assumes that when the gas inside the bubbles is heated to the ignition temperature, the reaction (on reaching the relevant degree of bubble compression R_0/R^*) proceeds instantaneously: there is an adiabatic explosion in a constant volume (constant bubble radius R^*) with an instantaneous pressure jump in the detonation products.

In the physical model considered, the basic system of equations [8] includes the laws of conservation for average characteristics (density ρ, pressure p, and velocity v) and a subsystem that describes the state of the medium and consists of the liquid-phase equations of state (the Tait equation), the gas-phase adiabat, and the Rayleigh equation of dynamics of "average single" bubbles. The collective effect of such bubbles on field characteristics is taken into account by the average pressure in the bubble medium, which substitutes for the pressure at infinity on the right side of the Rayleigh equation. This system has the form

$$\frac{d\rho}{dt} + \rho \operatorname{div} \boldsymbol{v} = 0, \qquad \frac{d\boldsymbol{v}}{dt} + \frac{\nabla p}{\rho} = 0, \qquad \rho = (1-k)\rho_{\text{liq}}, \qquad k = k_0\Big(\frac{R}{R_0}\Big)^3,$$

$$R\,\frac{d^2R}{dt^2} + \frac{3}{2}\Big(\frac{dR}{dt}\Big)^2 = \rho_{\text{liq}}^{-1}(p_{\text{g}} - p), \qquad p_{\text{g}} = \eta p_0\Big(\frac{R_0}{R}\Big)^{3\gamma}, \qquad p = B\Big[\Big(\frac{\rho}{\rho_0}\Big)^n - 1\Big].$$

Here k is the current gas-phase volume fraction, ρ_{liq} is the liquid-phase density, R is the current bubble radius, and B and n are constants in the Rayleigh equation. For a passive medium, we set $\eta = 1$, and for an active medium, $\eta = 1$ up to the moment $t = t^*$ when a bubble reaches the critical radius R^* corresponding to the moment of mixture ignition. If $R = R^*$, an adiabatic explosion occurs, the bubble volume remains unchanged, and the pressure p_{g} in the bubble increases instantaneously to $p_* = \rho_*(\gamma_* - 1)Q_{\text{expl}}$ (Q_{expl} is the heat of explosion and ρ_* is the density of the detonation products). In this case, the coefficient η and the adiabatic exponent γ change instantaneously by

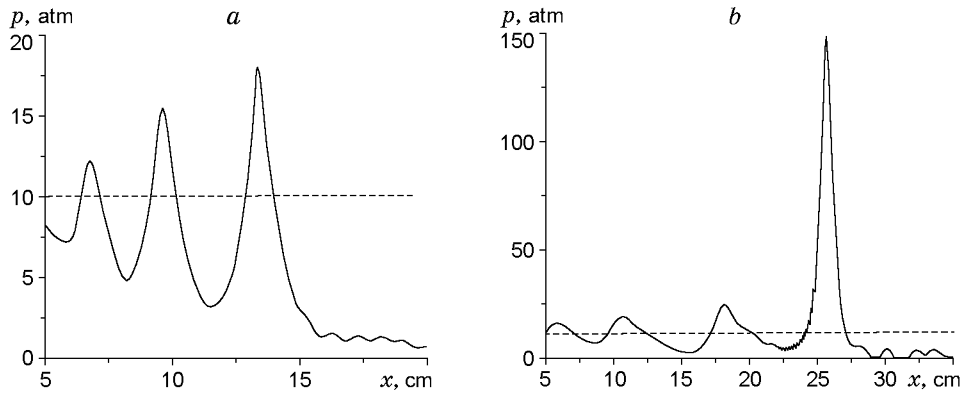

Fig. 2. Stationary shock-wave profiles in passive (a) and active (b) bubble media [the horizontal dashed lines correspond to the specified jumps in pressure $p(0)$ at the shock tube butt-end that initiate the wave process].

corresponding values. The adiabatic exponent becomes equal to that of the detonation products γ_*. After that, the process develops under new "initial" conditions without change of the mathematical model.

In the calculations, the liquid phase was water; in the case of a passive bubble medium, the gas-phase was an ideal gas with an adiabatic exponent of 1.4, and in case of an active medium, the gas-phase was a $2H_2 + O_2$ mixture. At the moment $t = 0$, the pressure $p(0)$ on the left shock-tube wall is constant.

In the numerical implementation of this model, we used the particle-in-cell method [9] modified for cavitating liquid flows in channels, which is given in detail in [10]. In the present paper, this method is modified for shock-wave processes in passive and chemically active bubble media. As was noted in [10], the main idea of this method is to split the initial unsteady system of equations into physical processes. The medium is simulated by a system of liquid particles which, at the current moment, coincides with an Euler grid cell. The calculation of each time step includes three stages. In the first stage, the effects associated with the transfer of an elementary cell are ignored (mass fluxes through the cell boundaries are absent). It is assumed that the liquid is accelerated only by the pressure gradient, and intermediate velocities are determined for particles in cells. At the second stage, the mass and momentum fluxes through the cell boundaries are calculated. At the final stage, the velocity, density, and pressure are determined for the next moment.

Formation of Shock Waves and Their Refraction by the Interface. The numerical scheme was tested by solving the one-dimensional problem of the formation of wave structures in a constant-area shock tube filled with a liquid containing gas bubbles. Typical structures of steady-state shock waves in the passive and active media are given in Fig. 2a and Fig. 2b, respectively. In both cases, $p(0) = 10$ atm, the gas-phase volume fraction was $k_0 = 0.01$, and the bubbles were of the same size ($R_0 = 0.1$ cm). For the indicated parameters, the establishment of a steady-state regime, for example, for a bubble detonation wave (maximum amplitude approximately 150 atm) is recorded at a distance of $x \approx 15$ cm from the left wall of the shock tube. A stationary wave in the passive medium (Fig. 2a) and the "tail" of a detonation wave (Fig. 2b) running in the bubble system with detonation products (passive system) have a classical oscillation structure. A system of precursors is generated ahead of the detonation leader and the shock-wave front.

To solve the problem of generation of acoustic radiation by bubble systems in a waveguide free of a gas-phase, it is necessary to analyze the structure and parameters of the waves refracted by the interface. In this case, the distance from the shock-tube butt-end to the interface is chosen such that a stationary wave profile has formed by the moment the wave collides with the interface. In the presence of an interface between the one-phase liquid and the bubble medium due to the abnormal compressibility of the latter and the large difference in acoustic impedances between these media, there may be an increase in the amplitudes of the shock wave or the bubble detonation wave when these waves are refracted by the interface (x_*), passing from the less dense medium to the denser medium. Results of numerical analysis of this effect (Fig. 3) show that the first oscillation of the shock wave due to refraction by the "passive medium–liquid" interface (Fig. 3a) and the first oscillation of the bubble detonation wave (Fig. 3b) are weaker than in the case of their reflection from a rigid wall. For example, according to the data of [4], in passive media, in the case of reflection from a wall, the wave pressure increases as a function of the volume fraction k_0

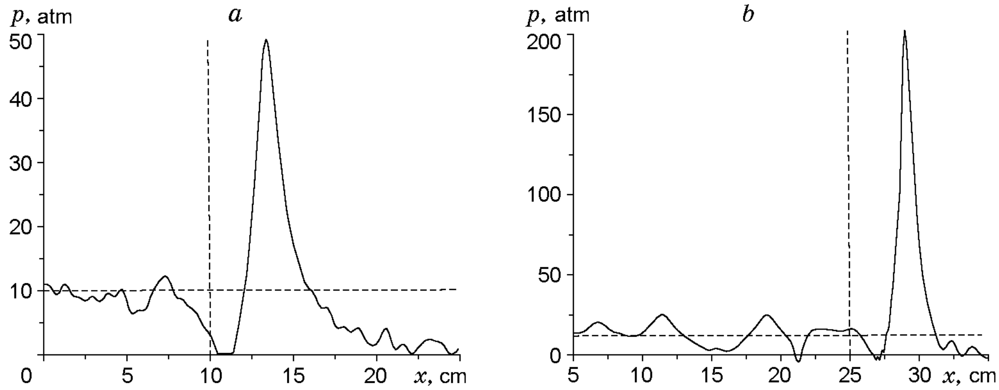

Fig. 3. Wave structure near the interfaces in the passive (a) and active (b) bubble media (the horizontal dashed lines correspond to jumps in pressure $p(0)$ specified at the shock-tube butt-end and the vertical dashed lines refer to the interfaces).

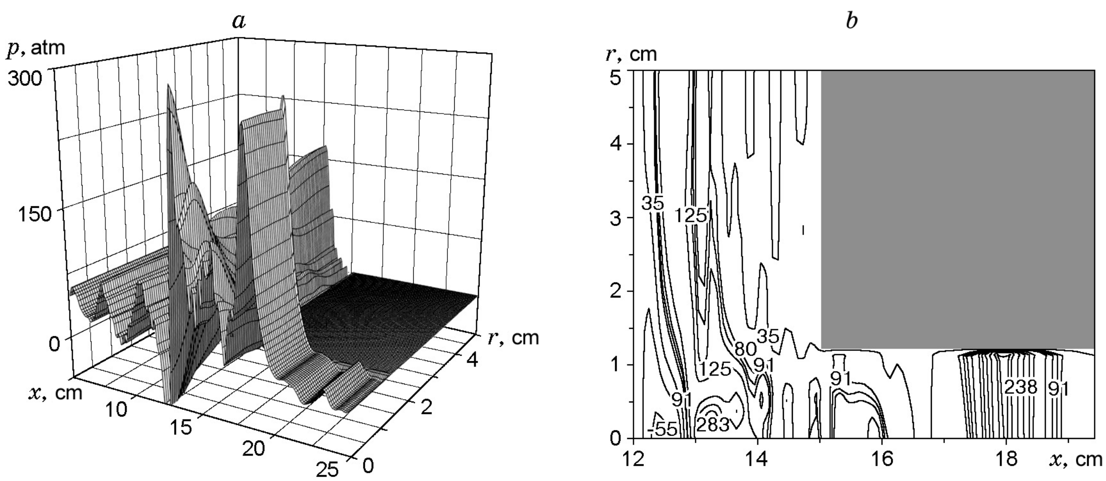

Fig. 4. Spatial pressure field (a) and isobars (b) at the moment $t = 200$ μsec in the shock tube and waveguide when the bubble detonation wave is reflected from a rigid wall with a hole ($k_0 = 0.01$ and $R_0 = 0.1$ cm; the shaded area is the rigid wall).

under the law $p_{\rm ref}/p(0) \approx 2 + 24.5k_0^{1/4}$ and increases by an order of magnitude under the conditions specified above. Figure 3b shows a profile of a refracted detonation wave whose "tail," propagating at lower velocity behind the detonation leader in a passive medium, has not reached the interface by the given moment. We note that ahead of the interface, the wave structure is distorted by the reflected waves.

Wave Generation in a Shock Tube with Sudden Changes of Cross Section. An analysis of numerical calculations showed that in the range of $k_0 = 0.005$–0.04 for $r_{\rm cyl}/R_{\rm ST} = 0.5$ and $R_0 = 0.1$ cm, the amplitudes of generated waves change insignificantly in both passive and active bubble media. Thus, for further analysis of the wave field structure in the shock tube, we use the value $k_0 = 0.01$.

Taking into account the complexity of the expected wave pattern in a shock tube with discontinuities in cross section (Fig. 1), we performed a preliminary analysis of the special features of the wave structure for two formulations: reflection from an annular wall in a shock-tube channel without a rod (see Fig. 1b) and expansion of a bubble detonation wave behind the butt-end of a coaxial rod of length of $L_{\rm cyl}$ in a "semi-finite" shock tube (see Fig. 1c) filled with an active bubble medium.

Reflection of a Detonation Wave from the Wall with a Hole. We consider the axisymmetric problem of the reflection of a bubble detonation wave in an annular tube (see Fig. 1b) from its butt-end in the form of a rigid wall with a hole for waveguide 5. Detonation is initiated over the entire tube cross section, and the wall coordinate x_w and the "bubble medium–liquid" interface ($x_* = 15$ cm) coincide with the waveguide mouth. Figure 4 shows

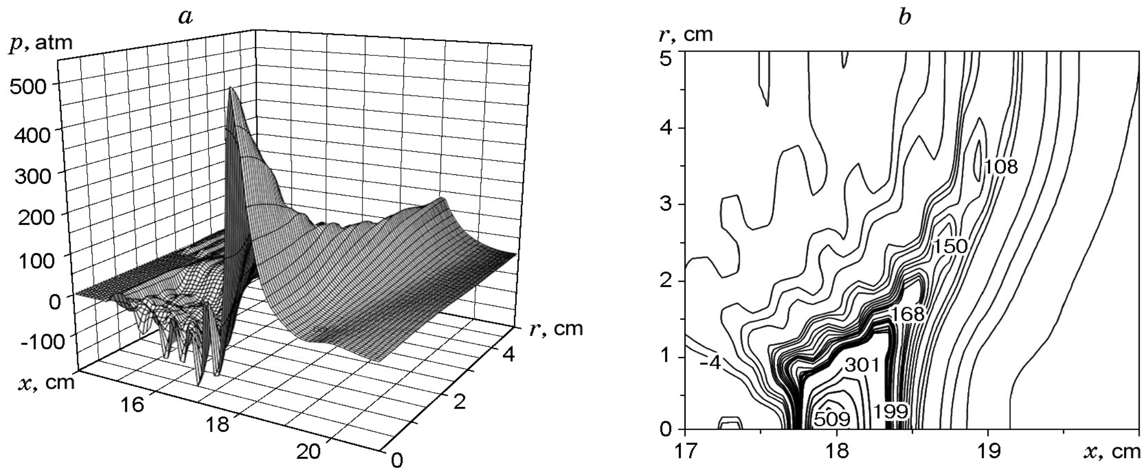

Fig. 5. Spatial pressure field (a) and isobars (b) at the moment $t = 225$ μsec in the shock tube for expansion and focusing of the bubble detonation wave.

a characteristic pressure field presented as a spatial distribution and a system of isobars in the shock tube and waveguide when the detonation wave is reflected from the wall. (In Figs. 4–6, the pressure on the isobars is given in atomospheres.) In Fig. 4, one can clearly see a pressure peak and axial focusing of the detonation wave reflected from the rigid wall (for the passive system). The maximum pressure in the focus spot reaches 300 atm, and the wave amplitude at the entrance to the waveguide ($r_{\rm out}/R_{\rm ST} = 0.25$) exceeds 200 atm; the pressure distribution over the waveguide radius is nearly uniform (Fig. 4a). The wall pressure decreases to the initial pressure in the incident wave by the given time. The reflected wave with the focus spot moves to the left in the passive medium. The shock wave generated in the waveguide is substantially one-dimensional, and its amplitude is almost similar to that of the detonation wave refracted by the interface and calculated in a one-dimensional formulation (see Fig. 3b).

Expansion and Focusing of the Bubble Detonation Wave. We consider the wave structure of the pressure field formed in the shock tube when the detonation wave expands in a bubble medium behind the butt-end of the coaxial rod (see Fig. 1c). When the wave leaves the annular channel 1 and enters section 3 of the semi-infinite shock tube behind the rod butt-end placed at a distance $x = L_{\rm cyl} = 15$ cm (Fig. 5a), a rarefaction region occurs. Obviously, at some distance from the rod butt-end, unloading is not so large and an amount of energy sufficient for bubble compression to the temperature of mixture ignition is preserved in the wave. Not only does the energy radiated by the exploding bubbles compensate for the losses by the wave but the wave can also be amplified by focusing, which is supported by calculation results (Fig. 5). In Fig. 5a, one can see a rarefaction region near the rod butt-end ($x = 15$–17.5 cm), in which a bubble detonation wave does not arise at this stage of the process. However, since the active mixture in the bubbles is preserved in this region, subsequent pulsations of these bubbles can lead to detonation.

In Fig. 5, the focus zone is well defined: at a certain distance from the rod butt-end, a Mach configuration forms, whose characteristic size exceeds 1 cm for a relatively small focusing radius ($r_{\rm cyl} = 2.5$ cm, $k_0 = 0.01$, $r_{\rm cyl}/R_{\rm ST} = 0.5$, and $R_0 = 0.1$ cm). The Mach configuration moves along the axis, and in the case considered (Fig. 5b), it is at a distance of 3 cm from the rod butt-end. We can assume that at a large distance, the pressure field will become uniform over the cross section, and the wave-field structure will tend to the "one-dimensional" variant. Obviously, there is some optimal geometry of the shock-tube cross section for which the amplitude of the resulting shock wave generated in the waveguide by the active bubble medium is maximal.

Shock Tube with Two "Discontinuities" in Cross Section. The formulation of the problem corresponds to Fig. 1a. Calculations show that in the general formulation, the wave radiated into the waveguide can be amplified using the effect of focusing and the wall effect, for example, by choosing the parameter L such that the Mach configuration forms near the interface (in the examples below, $L = 4$ cm).

The waveguide radius $r_{\rm out}$ can be used as a second control parameter, whose value must be chosen in accordance with the size of the Mach disk so that a one-dimensional wave forms in the waveguide channel. For example, if the waveguide radius is equal to the rod radius ($r_{\rm out} = r_{\rm cyl} = 2.5$ cm and $r_{\rm cyl}/R_{\rm ST} = 0.5$), the pulse amplitude in the waveguide control section 6 (see Fig. 1a) is 200 atm. When the exit channel radius $r_{\rm out}$ decreases to the size of the Mach stem (roughly 1 cm), the wave amplitude reaches 500 atm.

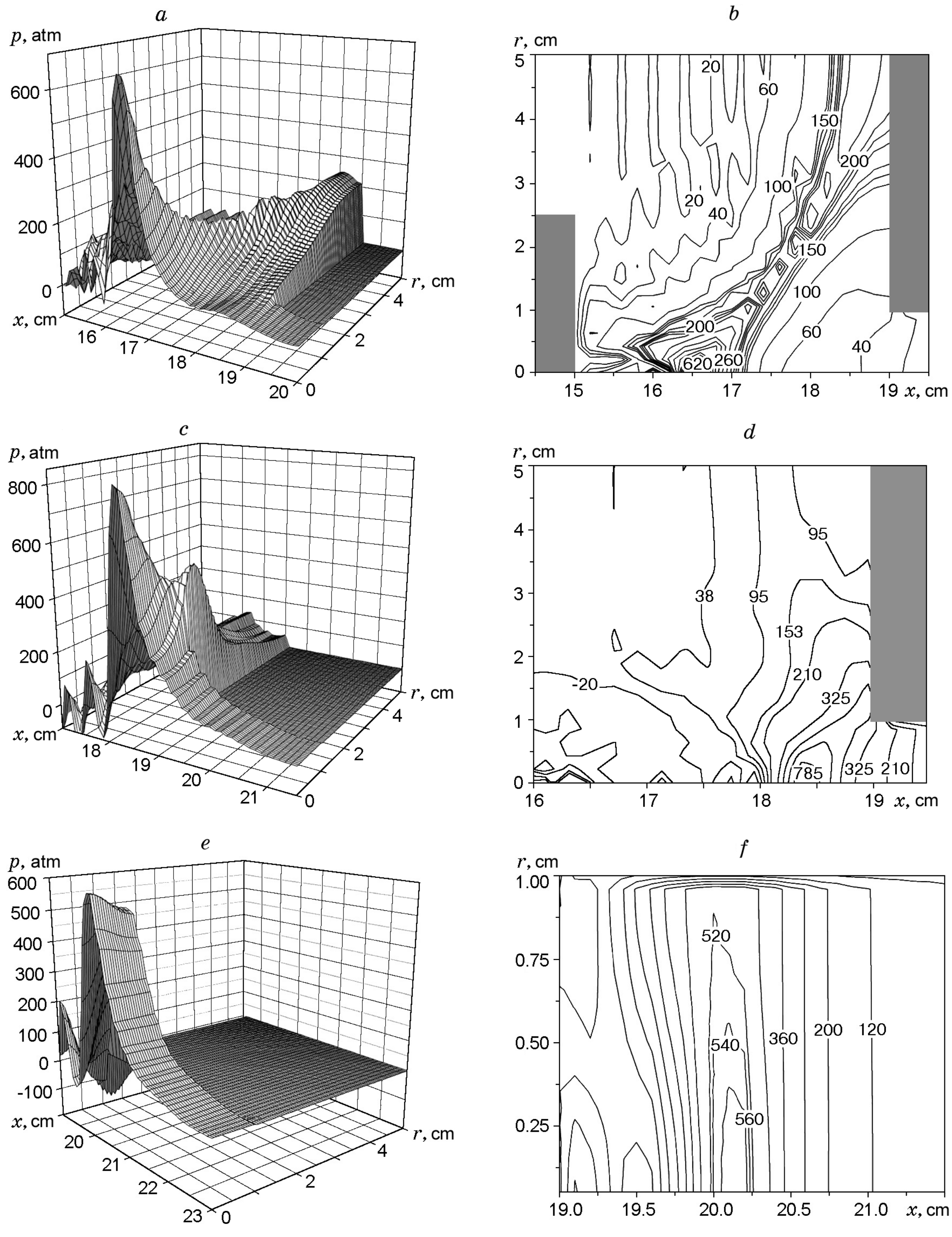

Fig. 6. Spatial pressure fields (a, c, and e) and isobars (b, d, and f) in the shock tube and waveguide for a bubble detonation wave focused and reflected from the wall (shaded areas correspond to the rod butt-end and the wall) for $t = 215$ (a and b), 225 (c and d), and 235 μsec (e and f).

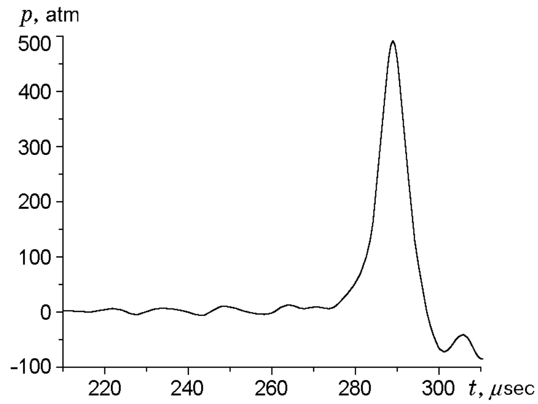

Fig. 7. Pressure-field evolution.

The calculation results shown in Fig. 6 support the above assumption on the possibility of considerable amplification of the wave generated in the waveguide by the active bubble medium as the bubble detonation wave propagates in the shock tube with "discontinuities" in section. At $t = 215$ μsec, the pressure at the center of the focus ($x_{\mathrm{f}} \approx 16.5$ cm) is in excess of 600 atm (Fig. 6a and b). In 10 μsec (Fig. 6c and d), the focus is displaced by 2 cm, and the amplitude in the focus increases to 800 atm (wall effect). Figure 6e and f shows the pressure distribution in the waveguide at $t = 235$ μsec. At this time, the wave crest in the waveguide is at a distance of roughly 1 cm from the interface and had a maximum amplitude of approximately 600 atm on the channel axis followed by a slight decrease (to 500 atm) at a distance of $r \approx 0.9$ cm from the axis (for $r_{\mathrm{out}} = 1$ cm). An abrupt decrease in pressure is observed in the millimeter zone at the waveguide wall.

Figure 7 shows the evolution of the characteristic pressure field in the waveguide section 6 (see Fig. 1a) starting from the moment of arrival of precursors. It is seen that in the hydrodynamic shock tube with "disocnti-nuities" in cross section filled with bubbles of an explosive gas mixture, a strong pressure pulse is generated in the waveguide upon excitation of a bubble detonation wave.

Conclusions. Refraction of shock waves and bubble detonation waves by the interface between a liquid with gas bubbles and a homogeneous liquid was analyzed numerically in a one-dimensional formulation. The analysis showed that the amplitudes of shock waves refracted into the homogeneous liquid for a one-dimensional formulation can far exceed those of incident waves but, at the same time, these amplitudes are much lower than in the case of reflection from the rigid wall.

The phenomenon of wave amplification in channels with "discontinuities" in cross sections was studied in an axisymmetric formulation. This phenomenon is observed when a wave is reflected from the wall or/and focused at the butt-end of a rigid rod aligned coaxially to the channel. In this configuration, the amplification is due to two-dimensional cumulation of the shock wave after it leaves the annular channel and reaches the rod butt-end. In the focusing spot, a Mach configuration forms. The pressure gradient across the channel radius is large. However, as the wave propagates over the active bubble medium along a shock tube of constant radius R_{ST}, the pressure is averaged over the cross section, and in the limit, it tends to the "pre-focus" value.

The geometrical characteristic of the shock tube allow one to control (in a certain range) the amplification coefficient and the position of the focusing spot. In particular, the wave can be focused in the vicinity of the rigid annular wall (in the region of passage through the interface to the waveguide) and amplified upon reflection. If the waveguide radius is equal to the height of the Mach stem, the amplitude of the radiated wave is maximal.

The work is supported by the Russian Foundation for Fundamental Research (Grant Nos. 00-02-18004a, 01-02-06426mac, and 00-15-96181).

REFERENCES

1. V. K. Kedrinskii, "Propagation of perturbations in a liquid containing gas bubbles," *J. Appl. Mech. Tech. Phys.*, **9**, No. 4, 370–376 (1968).
2. V. E. Nakoryakov, E. S. Vasserman, B. G. Pokusaev, and N. A. Pribaturin, "Amplification of the amplitude of pressure waves in a vapor–liquid medium with a bubbly structure," *Teplofiz. Vys. Temp.*, **32**, No. 3, 411–417 (1994).
3. V. E. Dontsov and V. E. Nakoryakov, "Propagation of shock waves in a porous medium saturated by a liquid with bubbles of a soluble gas," *J. Appl. Mech. Tech. Phys.*, **41**, No. 5, 845–854 (2000).
4. V. K. Kedrinskii, V. A. Vshivkov, G. I. Dudnikova, and Yu. I. Shokin, "Amplification of shock waves upon collision and focusing in bubble media," *Dokl. Ross. Akad. Nauk*, **361**, No. 1, 41–44 (1998).
5. V. K. Kedrinskii, Yu. I. Shokin, V. A. Vshivkov, et al., "Generation of shock waves in a liquid by spherical bubble clusters," *Dokl. Ross. Akad. Nauk*, **381**, No. 6, 773–776 (2001).
6. V. K. Kedrinskii, "The Iordansky–Kogarko–van Wijngaarden model: Shock wave and rarefaction wave interactions in bubbly media," *Appl. Sci. Res.*, **58**, Nos. 1/4, 115–130 (1997/1998).
7. V. K. Kedrinskii, P. A. Fomin, S. P. Taratuta, and A. A. Vasiliev, "Phase transition role in bubbly detonation problems," in: *Proc. 22nd Int. Symp. on Shock Waves* (London, July 18–23, 1999), Imperial College, London (1999), pp. 223–228.
8. V. K. Kedrinskii and F. N. Zamaraev, "Wave amplification in chemically active bubbly media," in: *Proc. 17th Int. Symp. on Shock Tube and Shock Waves* (Bethlehem, July 17–21, 1989), Lehigh Univ., Bethlehem (1989), pp. 51–62.
9. O. M. Belotserkovskii and Yu. M. Davydov, *The Particle-in-Cell Method in Gas Dynamics* [in Russian], Nauka, Moscow (1982).
10. S. P. Taratuta, I. V. Maslov, and A. S. Besov, "Two-dimensional simulation of cavitation flows in channels of arbitrary shapes," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], No. 117, Novosibirsk (2001), pp. 146–150.